Module 1

DFT properties

DFT

- Q1 Determine DFT of x(n)=[1 2 3 4]
- Solution:

$$\begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{vmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{vmatrix} 1 \\ 2 \\ 3 \\ 4 \end{vmatrix}$$

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$$= \begin{bmatrix} \mathbf{1+2+3+4} \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} \mathbf{10} \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

DFT

- Find IDFT of X(K)=[10 -2+2j -2 -2-2j]

$$\begin{vmatrix} \mathbf{x(0)} \\ x(1) \\ x(2) \\ x(3) \end{vmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{vmatrix} \mathbf{10} \\ -2+2j \\ -2 \\ -2-2j \end{vmatrix}$$

=

DFT

- Find IDFT of X(K)=[10 -2+2j -2 -2-2j]

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 10-2+2j-2-2-2j=4 \\ 10-2j-2+2+2j-2=8 \\ 10+2-2j-2+2+2j=12 \\ 10+2j+2+2-2j+2=16 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

DFT

Quiz:

1. Determine DFT of x(n)=[5,6,7,8] and find IDFT of result to verify answer
2. Determine x1(n)*x2(n)=y(n)

When x1(n)=[1,2,3,4]

X2(n)=[5,6,7,8]

Hint: 1)find X1(k) and X2(k)

2) Y(k)= X1(k).X2(k)

3) IDFT(Y(k))=y(n)

Circular frequency shift property

- Proof

➢ X(k)=$\sum_{n=0}^{N-1} x(n)W_N^{Kn}$

➢ DFT of x(n) W_N^{-ln}= $\sum_{n=0}^{N-1} x(n)W_N^{-ln}W_N^{Kn}$

= $\sum_{n=0}^{N-1} x(n)W_N^{(k-l)n}$

=(X(k-l))

DFT

- Circular frequency shift/ frequency translation
- ➢ DFT of x(n) W_N^{-ln}=X(k-l)
- ➢ DFT of x(n) W_N^{+ln}=X(k + l)
- ➢ DFT of x(n) $W_N^{Nn/2}$=X(k+$\frac{N}{2}$) and
- ➢ x(n) $W_N^{-Nn/2}$=X(k- $\frac{N}{2}$)
- ➢ Hence DFT of ($(-1)^n$x(n))=X(k - $\frac{N}{2}$)

DFT

- Example: Find DFT of x1(n)=[1, -2, 3, -4] using the DFT of x(n)=[1 2 3 4]

- Solution:
- Using, x1(n)=**how can we get this using x(n)**?

DFT

- Example: Find DFT of x1(n)=[1, -2, 3, -4] using the DFT of x(n)=[1 2 3 4]

- Solution:
- Using, x1(n)= $(-1)^n$x(n)
- Step 1….X(k)
- Step 2….. DFT of ($(-1)^n$x(n))=X(k - $\frac{N}{2}$)

DFT

- Example: Find DFT of x1(n)=[1, -2, 3, -4] using the DFT of x(n)=[1 2 3 4]

- Solution:
- Using, x1(n)= $(-1)^n$x(n)
- Step 1....X(k) = $\begin{vmatrix} \mathbf{10} \\ -2+2j \\ -2 \\ -2-2j \end{vmatrix}$
- Step 2..... DFT of ($(-1)^n$x(n))=X(k - $\frac{N}{2}$)
- =X(k -2)?????

DFT

- Example: Find DFT of x1(n)=[1, -2, 3, -4] using the DFT of
  x(n)=[1 2 3 4]

- Solution:
- Using, x1(n)= $(-1)^n$x(n)
- Step 1....X(k) = $\begin{vmatrix} \mathbf{10} \\ -2+2j \\ -2 \\ -2-2j \end{vmatrix}$
- Step 2..... DFT of ($(-1)^n$x(n))=X(k - $\frac{N}{2}$)= X(k -2)
- **=[-2, -2-2j, 10, -2+2j]**

DFT property:Circular time reversal / Circular folding

- Example: find DFT of x1(n)=[1, -2, 3, -4]
- Solution:Summary
- x1(n)= $(-1)^n$x(n)

- DFT of $(-1)^n$x(n)=X(k- $\frac{N}{2}$)=X(k -2)
- =[-2, -2-2j, 10, -2+2j]

DFT property:Circular time reversal / Circular folding

- Circular time reversal / Circular folding
- DFT of $x(-n)=X(-k)$

Example Find DFT of $x1(n)=[1,4,3,2]$

- Solution: $X(-k)=$

DFT property:Circular time reversal / Circular folding

Solved example:-

Using x(-n)=X(-k) or/ Circular time reversal property of DFT find DFT of x1(n)=[1,4,3,2].

Solution:-

$X(-k) =$ [10, -2-2j, -2, -2+2j] …………Answer

DFT: Symmetry property

- **Symmetry property**

  For a real valued sequence x(n), X(k)= X^*(N-k)

- **Solved Example**
- First five samples of 8-point DFT of sequence x(n) are given as below.
- X(K)=[0.5, 2+j, 3+2j, j, 3,…………….]

  Determine remaining samples of X(k)?

DFT: Symmetry property

- **Symmetry property**

  For a real valued sequence x(n), X(k)= X^*(N-k)
- **Solved Example**
- First five samples of DFT of x(n) are given as below. Determine value of remaining samples.
- X(K)=[0.5, 2+j, 3+2j, j, 3, __ ,___ ,___]
- X(5)= ?
- X(6)= ?
- X(7)=?

DFT: Symmetry property

- **Symmetry property**

  For a real valued sequence x(n), X(k)= X^*(N-k)
- **Solved Example**
- First five samples of DFT of x(n) are given as below. Determine value of remaining samples.
- X(K)=[0.5, 2+j, 3+2j, j, 3, __ ,___ ,___]
- X(5)= X^*(8-5) = X^*(3) =?
- X(6)= X^*(8-6)= X^*(2) = ?
- X(7)= X^*(8-7)= X^*(1) = ?

DFT: Symmetry property

- **Symmetry property**

  For a real valued sequence x(n), X(k)= X^*(N-k)
- **Solved Example**
- First five samples of DFT of x(n) are given as below. Determine value of remaining samples.
- X(K)=[0.5, 2+j, 3+2j, j, 3, __ ,___ ,___]
- X(5)= $X^*(8-5) = X^*(3)$ =-j
- X(6)= $X^*(8-6) = X^*(2)$ = 3-2j
- X(7)= $X^*(8-7) = X^*(1)$ = 2-j

DFT Properties

- **Solved Example:-** If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] find DFT of x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8]
- Find DFT of x(n).Also find the DFT of x1(n) and x2(n) using the DFT of x(n)

$$\begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{vmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{vmatrix} \mathbf{1+j5} \\ 2+j6 \\ 3+j7 \\ 4+j8 \end{vmatrix}$$

Solution:-X(k)=[???????]

$$X1(k)=\frac{X(k)+X^{*}(-k)}{2}=[?????]$$

$$X2(k)=\frac{X(k)-X^{*}(-k)}{2j}=[???????]$$

DFT Properties

- **Solved Example:-** If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] find DFT of x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8]
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Solution:-X(k)=[10+j26, -4, -2-j2, -j4]

$$X1(k)=\frac{X(k)+X^*(-k)}{2}=[?????]$$

$$X2(k)=\frac{X(k)-X^*(-k)}{2j}=[???????]$$

DFT Properties

- **Solved Example:-** If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] find DFT of x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8]
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Solution:-X(k)=[10+j26, -4, -2-j2, -j4]

$$X1(k)=\frac{X(k)+X^{*}(-k)}{2}=[10,\ -2+2j,\ -2,\ -2-2j]$$

$$X2(k)=\frac{X(k)-X^{*}(-k)}{2j}=[26,-2+2j,-4,-2-2j]$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- Let $g[n]$ and $h[n]$ be two length-N real sequences with $G[k]$ and $H[k]$ denoting their respective N-point DFTs
- These two N-point DFTs can be computed efficiently using a single N-point DFT
- Define a complex length-N sequence

$$x[n] = g[n] + j\,h[n]$$

Find DFT of g (4 point) and h (4 point) using DFT of X (4 point)

- Example - We compute the 4-point DFTs of the two real sequences $g[n]$ and $h[n]$ given below

$$\{g[n]\} = \{\underline{1} \;\; 2 \;\; 0 \;\; 1\}, \; \{h[n]\} = \{\underline{2} \;\; 2 \;\; 1 \;\; 1\}$$

- and $\{x[n]\} = \{g[n]\} + j\{h[n]\} = \{\underline{1+j2} \;\; 2+j2 \;\; j \;\; 1+j\}$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

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- Define a complex length-N sequence

$$x[n] = g[n] + j\,h[n]$$

- Let $X[k]$ denote the N-point DFT of $x[n]$

$$G[k] = \frac{1}{2}\{X[k] + X^*[\langle -k \rangle_N]\}$$

$$H[k] = \frac{1}{2j}\{X[k] - X^*[\langle -k \rangle_N]\}$$

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- **SOLUTION:-**

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j2 \\ 2+j2 \\ j \\ 1+j \end{bmatrix} = \ ?$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

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- **SOLUTION:-**

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j2 \\ 2+j2 \\ j \\ 1+j \end{bmatrix} = \begin{bmatrix} 4+j6 \\ 2 \\ -2 \\ j2 \end{bmatrix}$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- $X[k]$ is

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 4+j6 \\ 2 \\ -2 \\ j2 \end{bmatrix}$$

$X^*[k] =$?

$X^*[\langle -k \rangle_N] =$?

$$G[k] = \frac{1}{2}\{X[k] + X^*[\langle -k \rangle_N]\}$$

$$H[k] = \frac{1}{2j}\{X[k] - X^*[\langle -k \rangle_N]\}$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- (Let $X[k]$ denote the N-point DFT of $x[n]$

$$G[k] = \frac{1}{2}\{X[k] + X^*[\langle -k \rangle_N]\}$$

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- $X[k]$ *is*

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 4+j6 \\ 2 \\ -2 \\ j2 \end{bmatrix}$$

$X^*[k] = [4-j6 \quad 2 \quad -2 \quad -j2]$, $X^*[\langle -k \rangle_N] = [4-j6 \quad -j2 \quad -2 \quad 2]$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

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$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 4+j6 \\ 2 \\ -2 \\ j2 \end{bmatrix}$$

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$$X^*[k] = [4-j6 \quad 2 \quad -2 \quad -j2], \quad X^*[\langle -k \rangle_N] = [4-j6 \quad -j2 \quad -2 \quad 2]$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

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$X^*[k] = [4-j6 \;\; 2 \;\; -2 \;\; -j2]$, $X^*[\langle -k \rangle_N] = [4-j6 \;\; -j2 \;\; -2 \;\; 2]$

Answer….

Therefore

$$\{G[k]\} = \{4 \;\; 1-j \;\; -2 \;\; 1+j\}, \{H[k]\} = \{6 \;\; 1-j \;\; 0 \;\; 1+j\}$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT: Review

- Example - We compute the 4-point DFTs of the two real sequences $g[n]$ and $h[n]$ given below

$$\{g[n]\} = \{\underline{1} \;\; 2 \;\; 0 \;\; 1\}, \; \{h[n]\} = \{\underline{2} \;\; 2 \;\; 1 \;\; 1\}$$

- Then $\{x[n]\} = \{g[n]\} + j\{h[n]\} = \{\underline{1+j2} \;\; 2+j2 \;\; j \;\; 1+j\}$
- **SOLUTION:-**

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j2 \\ 2+j2 \\ j \\ 1+j \end{bmatrix} = \begin{bmatrix} 4+j6 \\ 2 \\ -2 \\ j2 \end{bmatrix}$$

- From the above

$$X^*[k] = [4-j6 \;\;\; 2 \;\; -2 \;\; -j2], \quad X^*[\langle 4-k\rangle_4] = [4-j6 \;\; -j2 \;\; -2 \;\; 2]$$

- Therefore

$$\{G[k]\} = \{4 \;\; 1-j \;\; -2 \;\; 1+j\}, \; \{H[k]\} = \{6 \;\; 1-j \;\; 0 \;\; 1+j\}$$ **Answer**

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- Let $g[n]$ and $h[n]$ be two length-N real sequences with $G[k]$ and $H[k]$ denoting their respective N-point DFTs
- These two N-point DFTs can be computed efficiently using a single N-point DFT
- Define a complex length-N sequence

$$x[n] = g[n] + j\,h[n]$$

- Let $X[k]$ denote the N-point DFT of $x[n]$

$$G[k] = \frac{1}{2}\{X[k] + X^*[\langle -k \rangle_N]\}$$

$$H[k] = \frac{1}{2j}\{X[k] - X^*[\langle -k \rangle_N]\}$$

- Note that for $0 \le k \le N-1$,

$$X^*[\langle -k \rangle_N] = X^*[\langle N-k \rangle_N]$$

N-Point DFTs of Two seqences Real Sequences ...using N point DFT

- Solved Example:-
- If x(n)=[1+j5, 2+j6, 3+j7, 4+j8]

  , x1(n)=[1, 2, 3, 4] and

  x2(n)=[5, 6, 7, 8]

  Find DFT of x(n).Also find the DFT of x1(n) and x2(n) using the DFT of x(n) and not otherwise.

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- Solved Example:-
- If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] , x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8].Find DFT of x(n).Also find the DFT of x1(n) and x2(n) using the DFT of x(n) and not otherwise.

Solution:-

X(k)=?

$$X1(k)=\frac{X(k)+X^{*}(-k)}{2})=?$$

$$X2(k)=\frac{X(k)-X^{*}(-k)}{2j})=?$$

N-Point DFTs of Two seqences Real Sequences …using N point DFT

- **Solved Example:-**If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] find DFT of x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8]Find DFT of x(n).Also find the DFT of x1(n) and x2(n) using the DFT of x(n)

$$\begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{vmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{vmatrix} 1+j5 \\ 2+j6 \\ 3+j7 \\ 4+j8 \end{vmatrix}$$

Solution:- X(k)=????

$$X1(k)=\frac{X(k)+X^*(-k)}{2}=$$

$$X2(k)=\frac{X(k)-X^*(-k)}{2j}=$$

DFT Properties

- **Solved Example:-** If x(n)=[1+j5, 2+j6, 3+j7, 4+j8] find DFT of x1(n)=[1, 2, 3, 4] and x2(n)=[5, 6, 7, 8]
- Find DFT of x(n).Also find the DFT of x1(n) and x2(n) using the DFT of x(n)

$$\begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{vmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{vmatrix} \mathbf{1+j5} \\ 2+j6 \\ 3+j7 \\ 4+j8 \end{vmatrix}$$

Solution:- X(k)=[10+j26, -4, -2-j2, -j4]

$$X1(k)=\frac{X(k)+X^{*}(-k)}{2}=?$$

$$X2(k)=\frac{X(k)-X^{*}(-k)}{2j}=?$$

2N-Point DFTs of a Real Sequences Using an N-Point DFT

- Let $v[n]$ be a length-N real sequence with a 2N-point DFT $V[k]$
- Define two length-N real sequences $g[n]$ and $h[n]$ as follows:

$$g[n] = v[2n],\ h[n] = v[2n+1],\ 0 \le n \le N$$

Example - Let us determine the 8-point DFT V[k] of the length-8 real sequence

$$\{v[n]\} = \{\underline{1}\ \ 2\ \ 2\ \ 2\ \ 0\ \ 1\ \ 1\ \ 1\}$$

We form two length-4 real sequences as follows

$$g[n] = v[2n] = \{\underline{1}\ \ 2\ \ 0\ \ 1\},\ h[n] = v[2n+1] = \{\underline{2}\ \ 2\ \ 1\ \ 1\}$$

2N-Point DFTs of a Real Sequences Using an N-Point DFT

- Let $v[n]$ be a length-N real sequence with a 2N-point DFT $V[k]$
- Define two length-N real sequences $g[n]$ and $h[n]$ as follows:

$$g[n] = v[2n],\ h[n] = v[2n+1],\ 0 \le n \le N$$

- Now

$$V[k] = \sum_{n=0}^{2N-1} v[n] W_{2N}^{nk} = \sum_{n=0}^{N-1} v[2n] W_{2N}^{2nk} + \sum_{n=0}^{N-1} v[2n+1] W_{2N}^{(2n+1)k}$$

$$= \sum_{n=0}^{N-1} g[n] W_N^{nk} + W_{2N}^{k} \sum_{n=0}^{N-1} h[n] W_N^{nk},\ 0 \le k \le 2N-1$$

- That is

$$V[k] = G[\langle k \rangle_N] + W_{2N}^{k} H[\langle k \rangle_N],\ \ 0 \le k \le 2N-1$$

2*N*-Point DFTs of a Real Sequences Using an *N*-Point DFT

- Example - Let us determine the 8-point DFT V[k] of the length-8 real sequence

$$\{v[n]\} = \{\underline{1}\ \ 2\ \ 2\ \ 2\ \ 0\ \ 1\ \ 1\ \ 1\}$$

- We form two length-4 real sequences as follows

$$g[n] = v[2n] = \{\underline{1}\ \ 2\ \ 0\ \ 1\},\ h[n] = v[2n+1] = \{\underline{2}\ \ 2\ \ 1\ \ 1\}$$

- Now

$$V[k] = G[\langle k\rangle_4] + W_8^k H[\langle k\rangle_4],\ \ 0 \le k \le 7$$

2N-Point DFTs of a Real Sequences Using an N-Point DFT

- Let $v[n]$ be a length-N real sequence with a 2N-point DFT $V[k]$
- Define two length-N real sequences $g[n]$ and $h[n]$ as follows:

$$g[n] = v[2n],\ h[n] = v[2n+1],\ 0 \le n \le N$$

- Let $G[k]$ and $H[k]$ denote their respective N-point DFTs
- Define a length-N complex sequence

$$\{x[n]\} = \{g[n]\} + j\{h[n]\}$$

with an N-point DFT $X[k]$

- Now $$V[k] = \sum_{n=0}^{2N-1} v[n] W_{2N}^{nk} = \sum_{n=0}^{N-1} v[2n] W_{2N}^{2nk} + \sum_{n=0}^{N-1} v[2n+1] W_{2N}^{(2n+1)k}$$

$$= \sum_{n=0}^{N-1} g[n] W_N^{nk} + W_{2N}^{k} \sum_{n=0}^{N-1} h[n] W_N^{nk},\ 0 \le k \le 2N-1$$

- That is $$V[k] = G[\langle k \rangle_N] + W_{2N}^{k} H[\langle k \rangle_N],\ \ 0 \le k \le 2N-1$$

Relation Between DFT and z-Transforms

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x[n] e^{-j\Omega}\Big|_{\Omega=\frac{2\pi}{N}k}$$

$$= X(\Omega)\Big|_{\Omega=\frac{2\pi}{N}k}$$

The DFT of $x[n]$ is its DTFT evaluated at N equally spaced points in the range $[0,2\pi)$.

For a sequence for which both the DTFT and the z-transform exist,

we see that:

$$X(k) = X(z)\Big|_{z=e^{j\frac{2\pi}{N}k}}$$

DFT

- Long data filtering method
- Overlap add
- Overlap save

DFT

- Overlap add method
- X(n)=[1, 2, 3, 4, 5, 6, 7, 8] h(n)=[1, 2]
- Nx=8, Nh=2, required Nx=mNh so m=4
- If Nx ≠ mNh ,padd zeros
- Split X(n) in 4 block h(n)=[1,2,0,0]
- X1(n)=[1,2,0,0]
- X2(n)=[3,4,0,0]
- X3(n)=[5,6,0,0]
- X4(n)=[7,8,0,0]
- Find
- y1(n)=x1(n)*h(n)=[1,4,4,0]
- Y2(n)=x2(n)*h(n)=[3,10,8,0]
- Y3(n)=x3(n)*h(n)=[5,16,12,0]
- Y4(n)=x4(n)*h(n)=[7,22,16,0]

DFT

1	4	4	0						
		3	10	8	0				
				5	16	12	0		
						7	22	16	0
1	4	7	10	13	16	19	22	16	0 DISCARD

Y(N)=[1,4,7,10,13,16,19,22,16]